A Model for High Resolution TerraSAR-X Images of Natural Scenes

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Abstract
In the last year, the TerraSAR-X sensor began to provide a great amount of data, thus increasing the possibility to extract valuable information from SAR images. Indeed, these high resolution images are increasingly more textured and present many details previously masked due to the low resolution of the available sensors. This certainly represents an advance for remote sensing applications, but, anyway, the interpretation of these images requires careful attention. As an example, in classification and segmentation issues the increasing in texture of the images is not so unmistakably good news, because, if adequate models for the new data are not introduced, the results may not be as reliable as they could.

In this paper we present the first steps toward the modeling of high resolution SAR images relevant to natural scenes: these are the first results obtained in the framework of the MTH0183 project approved after the TerraSAR-X AO. We use fractal models to account for the geometric description of the observed surface and effective scattering models to evaluate a closed form expression for the radar cross section of a fractal surface: in fact, the combined use of these fractal geometrical and electromagnetic models allows to obtain an effective data modeling. Hence, in this work we present a mathematical model allowing the evaluation in closed form of the structure function and of the power density spectrum of the image relevant to a natural scene, in case of small slopes for the observed scene. Furthermore, a numerical study applied to simulated data and aimed to the validation of the above-mentioned theoretical results is provided.
Finally, the obtained theoretical and experimental results are effectively used for the modeling of actual TerraSAR-X high resolution, amplitude only, SAR images. In particular, this analysis is performed on the Vesuvio volcano area close to Naples, Italy.

1. INTRODUCTION

In the last year the TerraSAR-X sensor provided a huge amount of high resolution images of the earth surface, whose increasingly rich texture allows, at least in principle, the extraction of a great number of previously unavailable geophysical information. However, their interpretation is neither straightforward nor simpler with respect to the case of low resolution data, introducing, on the contrary, new problems and challenges.

This is particularly true in the case of SAR images of urban areas, where many new features begin to appear as the resolution increase: this calls for an effective direct modeling of the data in support of information extraction. Anyway, thanks to the man-made character of the scene, these images are often speckle-free and the information of interest is related to deterministic and punctual characteristics of the scene: in this case the typical goals are the identification of buildings and their discrimination from streets or vegetated areas, the retrieving of the height of the buildings or the width of the street and so on. Conversely, in this paper we deal with high resolution images of natural scenes: we are in presence of speckle and value-added information is no longer found in deterministic and punctual features of the image but much more in its stochastic and global properties. Thus, any image processing technique aimed to the extraction of significant geophysical
parameters or to segmentation and classification issues should be based on a sound modeling of these kind of image characteristics.

In this paper, we present a model for the imaging process of natural profiles recently developed by the authors [1]. In particular, use is made of fractal models to account for the geometrical characterization of the imaged profile: in fact, this models are widely recognized as the best ones for the description of natural surfaces, in terms of few physically-based fractal parameters [2]. Furthermore, to account for the interaction of the electromagnetic field with the surface use is made of sound electromagnetic scattering models specifically developed for the case of a fractal surface [3]. In fact, theoretical and experimental studies show that the use of fractal-based scattering models strongly improves the accuracy in the evaluation of the backscattered field. Starting from this existing models we computed in closed form the power density spectrum and the structure function of the image of a fractal profile, if a small slope regime can be assumed. In particular, it’s important to note that the computed functions come to be dependent on the fractal parameters of the original profile and on the sensor resolution, allowing a better understanding of the role of resolution in the imaging process. Moreover, let us note that, for the moment, we focus on the one-dimensional case of a fractal profile: when (two-dimensional) images are of interest (i.e. in Section 4 of this paper) the results presented here can be used without modifications only for range-cuts, due to the particular acquisition geometry of the sensor. In Section 2 the fundamentals of the imaging model are provided.

In Section 3 a validation of the theoretical results is presented. It is based on a numerical framework [1] which, combining all the above-mentioned models, allows the comparison of the theoretical and actual spectra of the images relevant to a synthetic fractal profile with known fractal parameters. The results presented here clearly show the effectiveness of the proposed model in describing the imaging of fractal small slope profiles.

Finally, in Section 4 an analysis of actual TerraSAR-X amplitude only images is presented. The case study is relevant to the area of the Vesuvio volcano close to Naples, Italy, and the used image is the amplitude one obtained from an SSC stripmap acquisition of the zone. A low resolution Digital Elevation Model (DEM) of the same area, obtained from contour line interpolation, is available to the authors. The histograms, spectra and variograms of the same zone of the DEM and of the SAR image are estimated and compared with theoretical results. These preliminary results seem to be encouraging: in fact, the proposed model presents a good fit with the actual data despite the non-small slope character of the zone of interest and the presence of a significant speckle noise, whose contribution is not actually included in the theoretical model. Moreover, let us note that the available DEM is strongly vitiated not only by the low resolution but, mainly, by the elaboration technique used to obtain it from the contour lines. However, useful observations on spectral and variogram analysis of TerraSAR-X amplitude images are reported. Note that future developments of the presented theoretical and experimental results will allow a direct estimation of the parameters of the imaged surface from its radar image.

2. FRACTAL IMAGING MODEL

In this paper the observed profile is modeled as a fractional Brownian motion process [2], [3]. It is a continuous, but non-differentiable non-stationary process and can be defined through the probability density function of its increments. In fact, a stochastic process \( z(x) \) is an fBm profile if, for every \( x \) and \( x' \), it satisfies the following relation:

\[
Pr\{z(x) - z(x') < \xi\} = \frac{1}{\sqrt{2\pi st^n}} \int_{-\infty}^{\xi} \exp\left(-\frac{\xi^2}{2s^2 t^n}\right) d\xi
\]  

(2.1)
where \( \tau = |x - x'| \), \( H \) is the Hurst coefficient \((0 < H < 1)\) and \( s [m^{(1 - H)}] \) is the standard deviation of the profile increments at unitary distance.

The structure function \( V(\tau) \) (whose plot is termed the variogram) is defined as the mean square increment of elevation points placed at distance \( \tau \) and for an fBm profile can be evaluated in terms of the parameters \( H \) and \( s \) as:

\[
V(\tau) = s^2 \tau^{2H}. \tag{2.2}
\]

It has been demonstrated [3] that the power density spectrum \( S(k) \) of an isotropic fBm one-dimensional process exhibits appropriate power-law behaviors provided by:

\[
S(k) = s^2 \frac{\pi H}{\cos(\pi H)} \frac{1}{\Gamma(1 - 2H)}|k|^{-(2H+1)}, \tag{2.3}
\]

\( \Gamma \) being the Gamma function.

Note that both the structure function (2.2) and the power spectrum (2.3) of an fBm profile exhibit a linear behavior in a log - log plane, allowing the use of linear regression techniques for the retrieving of the fractal parameters of the profile from measured data.

In Ref. [1] the authors presented the rationale of an imaging model based on the fBm description of the profile. In particular, the key hypothesis of the model is the possibility to assume a small slope regime for the considered profile at least at resolution scale: in this case, the scattering behavior results to be linearly related to the slope of the imaged profile evaluated at resolution scale. In fact, in this case we can assume a McLaurin first order expansion of the image intensity for small values of the derivative \( p(x) \) and obtain:

\[
i(x) \equiv a_0 + a_1 p(x). \tag{2.4}\]

Introducing an adequate model for the derivative of the fBm and making physical considerations on the role of system resolution in the imaging process the authors evaluated in closed form the structure function \( V_I \) and the power density spectrum \( S_I \) relevant to the image [1]:

\[
V_I(\tau; \Delta x) = a_1^2 s^2 \Delta x^{2H-2} \left[ 2 - \left( \frac{|\tau|}{\Delta x} + 1 \right)^{2H} + 2 \left( \frac{|\tau|}{\Delta x} - 1 \right)^{2H} \right] \tag{2.5}
\]

\[
S_I(k; \Delta x) = 2a_1^2 s^2 \Delta x^{-2} \Gamma(1 + 2H) \sin(\pi H) \left( 1 - \cos(\pi \Delta x) \right) |k|^{-(2H+1)} \tag{2.6}
\]

where \( \Delta x \) is the system resolution. Equations (2.5) and (2.6), in the limit of \( \frac{\tau}{\Delta x} >> 1 \) and \( k \Delta x << 2\pi \) respectively, take the relevant forms:

\[
V_I(\tau; \Delta x) = 2a_1^2 s^2 \left[ \Delta x^{2H-2} - H(2H - 1)|\tau|^{2H-2} \right] \tag{2.7}
\]

\[
S_I(k) = a_1^2 s^2 \Gamma(1 + 2H) \sin(\pi H) |k|^{-(2H+1)} \tag{2.8}
\]
The plots of (2.5) and (2.6) for $H=0.75$, $s=0.1\, m^{0.25}$, $\alpha=10$ and $\Delta x=3\, m$ are reported in Fig. 1 and Fig. 2 respectively and are compared with the variogram and the power density spectrum of the profile.

![Fig. 1 Variograms of the image (full line) and of the profile.](image1)

![Fig. 2 Spectra of the image (full line) and of the profile.](image2)

Note that from Eq. (2.8) we can conclude that in the asymptotic region the power density spectrum of the image presents a power law behavior and the fractal parameters of the imaged profile can be still estimated via a linear regression on a log – log plane.

3. NUMERICAL RESULTS

In the present section we provide numerical results supporting the theoretical model introduced in the previous section. The numerical setup is based on a fractal SPM scattering model [3] for the evaluation of the field backscattered from the considered profile. In fact, it has been demonstrated [3] that the use of fractal electromagnetic models strongly improves the accuracy in the evaluation of the received signal. In fact, the use of heuristic scattering functions, which is very common in the literature, can lead to very inaccurate results and is not useful in case of fractal surfaces. In the following we summarize the key steps for the synthesis of the numerical framework.

First of all, we generate a fractal fBm profile using the Weierstrass-Mandelbrot (WM) function: in fact, under some hypothesis, the WM effectively approximates an fBm profile [3]. Once the profile is synthesized, we evaluate the backscattered signal via the SPM fractal scattering model [2]. The geometrical model used in this section is based on the assumption that the observed profile shows the same fractal parameters at all the scales of interest: in particular, at scales greater and lower of the resolution one.

To validate the results presented in the previous sections, we estimated the power density spectra of the profile and of the backscattered signal. Note that particular care has to be paid to the estimation of these power law spectra, because they are subject to extreme leakage and high variance problems. Hence, for the evaluation of all the spectra presented in the paper use is made of a Capon one-dimensional estimator [4]. In Fig. 3 significant numerical results are presented, where $H=0.95$, $s=0.01 \, m^{0.05}$, the sensor look angle is set to 10° and $\Delta x=1\, m$. A very good fit between the curves relevant to theoretical and numerical results can be observed. Note that the image spectrum shows a linear behavior in the log – log plane due to the fact that we are estimating only where the asymptotic expression of Eq. (2.8) can be assumed to be valid.

The particular choice of the parameters, both those relevant to the surface ($H$ and $s$) and to the system ($\Delta x$), is responsible for the validity of the small slope hypothesis on which the proposed imaging model is based. In fact, the fractal parameters account for variations in the roughness of the observed profile and, obviously, as the roughness increase also the slopes tend to increase. On the other side, the sensor resolution dictates the scale at which we observe the slope of the profile and larger this scale is larger the profile slopes will be [1]. Anyway, a detailed analysis of this phenomena is behind the scope of this paper.
4. ANALYSIS OF ACTUAL TERRASAR-X DATA

In this section we present significant results relevant to the analysis of actual TerraSAR-X data. In particular, we focus on the analysis of a case study: the area of interest is the Vesuvio volcano zone close to Naples, Italy.

As for the data we consider here an amplitude image obtained from an SSC standard stripmap product with a 1.5 x 3 m² pixel spacing in azimuth – ground range. An image of the area is provided in Fig. 4, where a rotation has been applied to the original data (in all the following images near range is on the bottom) and a 4x4 multi-look has been performed. In Fig. 5 a three-dimensional visualization of the DEM of approximately the same zone to which is relevant the image is shown: in this case the resolution is 20 x 20 m² and a vertical illumination is present to obtain the shaded representation. The DEM is obtained through interpolation of contour lines and this is evident observing the unnatural circular features surrounding the volcano. Obviously, the use of a higher resolution, better synthesized DEM would increase the accuracy of the elaborations presented hereafter.
Our analysis is focused on a limited subset of the image in Fig. 5: we selected only the volcano zone, discarding the surrounding built-up areas. In particular the considered zone is shown in Fig. 6 and 7 for the radar image (2 x 2 multi-look with obtained pixel spacing of 3 x 6 m²) and the DEM respectively.

On this area we estimated the function structure and the power density spectrum on approximately the same one-dimensional range cut. In fact, as we mentioned in Section 1, the results found for the one-dimensional profile apply without modifications to range cuts on two-dimensional images. The obtained log – log plots are presented in Fig. 8, 9, 10 and 11.
Fig. 8 Variogram relevant to the image of Fig. 6.

Fig. 9 Variogram relevant to the DEM of Fig. 7.
Fig. 10 Power density spectrum relevant to the image of Fig. 6.

Fig. 11 Power density spectrum relevant to the DEM of Fig. 7.
Some observations on these results are now in order:

- the difference in resolution between the DEM and the image is evident looking to the domain of the presented plots: in fact, those relevant to the DEM lack the higher frequency/ lower scale components if compared to those relevant to the image;
- the oscillating behavior of the DEM variogram is due to the particular technique used to generate it from contour curves;
- the higher frequency/ lower scale behavior of the plots relevant to the image is strongly affected by the speckle noise: in fact, at least in principle, looking at these plots it is possible to determine the number of looks needed to effectively reduce this effect.
- in the lower frequency/ higher scale region the estimation procedure cannot be considered reliable being based only on few samples;
- in this case study the validity of the small slope hypothesis is questionable being the image relevant to a very rough terrain;
- as can be seen from Fig. 6, geometrical distortions, as foreshortening and layover, are also present in the considered image: this non linear effects, for the moment, are not included in the proposed theoretical model.

However, looking to Fig. 10 and 11 and comparing them with Fig. 3 is possible to appreciate approximately the same general behavior, at least for the mid-frequency region of the presented plots.

In conclusion this preliminary results seem to suggest that the development of a technique for the retrieving of the fractal parameters of the observed surface from its image based on the proposed model is possible. The next steps toward this goal lies in an adequate treatment of the speckle noise and on the development of more accurate spectral-based estimation techniques.

5. CONCLUSIONS

In this paper a mathematical model for the imaging of small slope fractal profiles was presented. A numerical validation of the model was also proposed and useful observations on its limits of validity were provided.

The analysis of actual TerraSAR-X data relevant to the zone of the Vesuvio volcano demonstrates the effectiveness and the potentialities of the presented model in the modeling of high resolution amplitude images. Future developments, concerning mainly an accurate modeling of the speckle noise, will allow a model-based retrieving of significant geophysical parameters of the observed surface from its SAR image.

6. REFERENCES